Problem 1 **CMWMC 2025, Relay Round, Set 1/4**

Define $\gcd(a, b)$ to be the largest common divisor of integers a and b. Likewise, define $\operatorname{lcm}(a, b)$ to be the least common multiple of integers a and b. Suppose x,y are positive integers such that $x + y = 45$. Compute the maximum value of $\operatorname{lcm}(x, y) + \gcd(x, y)$.

Problem 2 **CMWMC 2025, Relay Round, Set 1/4**

Let T denote the answer to the previous problem. Let N be the remainder when T is divided by 100. Compute the number of ordered pairs of **non-negative integers** (x, y) such that

$$20x + 25y = 100N$$

Problem 3 **CMWMC 2025, Relay Round, Set 1/4**

A positive integer x in base 10 has three digits. When x is expressed in base T, it also has three digits. When x is expressed in base $2T$, it yet again has three digits. How many possible values of x are there?

Problem 1 **CMWMC 2025, Relay Round, Set 2/4**

Henry has a weight scale that is slightly broken. When he places n fruits on the scale, it calculates the total weight of the fruits, but adds on an amount proportional to the number of fruits before reporting. Specifically, it will add nk, where k is always constant and unknown to Henry.

Henry places 2 apples and 3 bananas on the scale and the scale measures 60 g. He then places 4 apples and 5 bananas and the scale measures 110 g. What will the scale report when 5 apples and 5 bananas are placed, in grams? (All apples are identical and all bananas are identical).

Problem 2 **CMWMC 2025, Relay Round, Set 2/4**

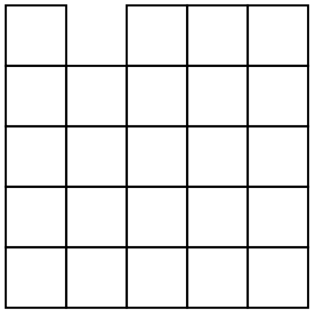
Krish is walking up an escalator. The escalator is partially functional; it will start by moving 2 stairs per second for five seconds, then stop moving for five seconds, and this cycle continues. Krish walks at a constant rate of 3 stairs per second. How many seconds will it take Krish to climb a T-stair escalator if he is always walking?

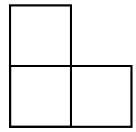
Problem 3 **CMWMC 2025, Relay Round, Set 2/4**

Kyle draws the circle $x^2 + (y - T)^2 = 16$ and the parabola $y = x^2 + k$. Find the **minimum** value of k such that these graphs intersect.

Problem 1 **CMWMC 2025, Relay Round, Set 3/4**

Colin is tiling the following 5×5 grid with the second square removed. He begins placing L shaped triomino tiles (shown below to the right) in any orientation such that each tile covers exactly 3 squares of the grid and there is no overlap among any tiles. What is the maximum number of tiles he can place?





L-shaped triomino

Problem 2 **CMWMC 2025, Relay Round, Set 3/4**

Let T denote the answer to the previous problem. Let $K = T + 4$. Sam and Jenny are trying to decide who gets to eat the last Subway Cookie with a game of rock paper scissors. Sam's strategy is to pick rock with probability $\frac{2}{K}$, paper with probability $\frac{6}{K}$, and scissors with probability $\frac{K-8}{K}$. Hearing this, Jenny decides her strategy to independently pick rock with probability $\frac{K-8}{K}$, paper with probability $\frac{2}{K}$, and scissors with probability $\frac{6}{K}$.

The probability that Sam wins in one game is $\frac{N}{K^2}$. What is N?

Problem 3 **CMWMC 2025, Relay Round, Set 3/4**

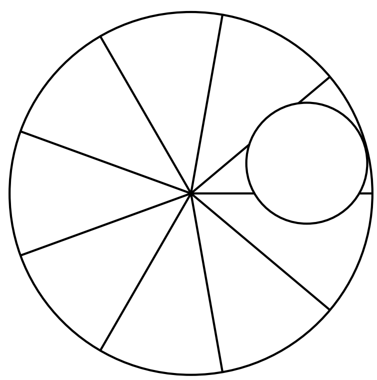
Let T denote the answer to the previous problem. Michael is building four towers with his limitless supply of blocks. His towers are initially T^2, $T(T+1)$, $T(T+1)$, and $(T+1)^2$ blocks high, respectively. We consider a "*move*" to be choosing three distinct towers, and placing 1 block on top of each of them. What is the minimum amount of moves required in order for the heights of all the towers to be equal?

Problem 1 **CMWMC 2025, Relay Round, Set 4/4**

Let convex pentagon $ABCDE$ have sides $AB = 3, BC = 7, CD = 5, DE = 7$, and $EA = 4$. Furthermore, angles A, C, D are right angles. What is the area of this pentagon?

Problem 2 **CMWMC 2025, Relay Round, Set 4/4**

Let T denote the answer to the previous problem. On a piece of paper, Dustin draws a circle of radius 6 and draws lines to split the circle into T equal sectors. April has a circular sticker of radius 2 and places it on the drawing such that it is internally tangent to the bigger circle. After this, x sectors are **not** fully visible. Find the **minimum** possible value of x. (As an example, if $T = 9$, the below sticker placement leaves 3 regions not fully visible.)



Problem 3 **CMWMC 2025, Relay Round, Set 4/4**

Let T denote the answer to the previous problem. A spherical yoga ball has radius $2T$ inches. Michael takes a hose and fills the yoga ball with some water. As a result, the height of the water in the ball is T inches (as illustrated on the left). Michael then adds Z more cubic inches of water to completely fill the yoga ball. Find Z. (Note: A spherical sector, as pictured on the right, has a volume $V = \dfrac{2\pi r^2 h}{3}$, where r is the radius of the sphere and h is the height of the spherical "cap".)