CMWMC 2025, Guts Round, Set 1/7

1. How many positive factors of 2025 are either perfect squares or perfect cubes?
2. How many ordered pairs of non-negative integers (a, b) satisfy $\sqrt{a} + \sqrt{b} = \sqrt{a + b + 104}$?
3. In unit square $ABCD$, let E be on BC and F be on AD such that triangles AEF and ECF are congruent and isosceles, with bases of AF and CE. What is the length of the path of $AE + EF + FC$?

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4. For a three-letter string, define the "score" to be the position of the word in an alphabetically-sorted list of all three-letter strings. For example, "aaa" would have a score of 1, "aad" would have a score of 4, and "zzz" would have a score of 26^3. My friend wrote the word "dig" on a napkin and asked me to compute the score, but a jester sneakily erased a random letter and wrote a **different** random letter in its place (eg. "dig" to "dxg"). What is the probability that the score went up?
5. The three sets A, B, and C each have 400 elements. Each intersection of two sets has 150 elements. The intersection of all three sets has 80 elements. How many elements are in at least one set?
6. Let OA_1 be a line segment with side length 1. From the point A_i, we generate the next point A_{i+1} in the following manner: we construct A_{i+1} such that $A_{i+1}A_i \perp A_iO$ and $\overline{A_{i+1}A_i} = i + 1$. Find the length of OA_{10}.

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7. Define the function $f(n)$ to take the largest integer k such that $n^k \mid n!$. Find

$$\sum_{i=2}^{6} f(i!).$$

8. I have a digital gate (number machine) Z that accepts three inputs and gives one output. Each input is allowed to be either 0 or 1. The output is a 1 if and only if exactly two inputs are 1. Otherwise, the output is 0.

   I have seven inputs a, b, c, d, e, f, g. Inputs a, b, c feed into a Z gate and e, f, g feed into another Z gate. The two outputs and d are fed as inputs to a third Z gate, which gives a final output.

   How many input strings for $abcdefg$ are there such that the final output is a 1?

9. Alice and Bob received a unit circle cookie from their teacher for being such excellent students. However, they aren't so excellent at splitting cookies. To share the cookie, they each pick a point on the circle's edge uniformly at random and cut a straight line from one point to the other. What is the expected difference in perimeters between the big and small cookie chunks?

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10. Find the sum

$$\sum_{n=1}^{\infty} \frac{3n}{(n+1)!}$$

11. I start with the number 32. I have three possible operations to transform this number into another number: subtract 3, add 4, and divide by 5 (only possible if the number is divisible by 5). What is the least number of operations needed to get to 0?

12. Jerry starts at 0 on the real number line. He tosses a fair coin exactly 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{m}{n}$, where m and n are relatively prime positive integers. (For example, he succeeds if his sequence of tosses is HTHHHHHH since he reaches 4 after the 6th flip.)

    What is $n + m$?

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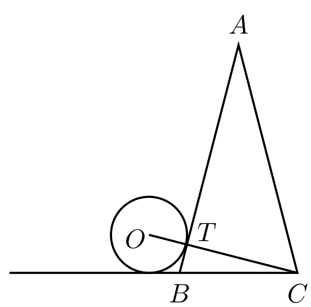
13. Isosceles triangle ABC has base BC length $2\sqrt{3}$ and side length $4\sqrt{3}$. Circle ω with center O is tangent to AB and line BC. Let r be the radius of ω. Let T be the point of tangency of ω and AB. If O, T, and C are collinear and $r^2 = \frac{a}{b}$, where a and b are relatively prime positive integers, find $a + b$.



14. Let a_n denote the positive integer whose digits are n ones in base ten. For example, $a_3 = 111$. Let S be the sum of the digits of $(3a_{2025} + 1)^2$ and T be the sum of the digits of $(6a_{2025} + 1)^2$. Compute $S + T$.

15. If x and y are positive real numbers such that

$$\frac{25}{x} + \frac{2025}{y} = 1,$$

compute the minimum possible value of $x + y$.

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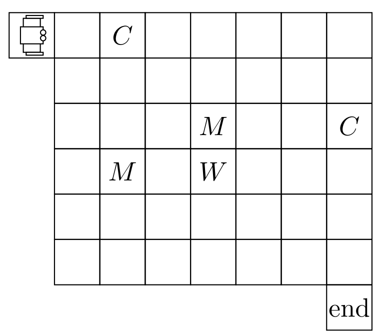
16. Consider a triangle $\triangle ABC$ with $BC = 17$, $CA = 15$, and $AB = 8$. Suppose the midpoint of BC is M and the interior bisector of $\angle A$ intersects the perpendicular bisector of BC at P. Find the area of $\triangle AMP$.

17. On a 6×7 grid, a robot starts at a cell immediately west of the northwest corner, facing east. The end cell is the one immediately south of the southeast corner. Natalie has programmed the robot to move at each cell in the following manner:

    - if the current cell has no letter, move to the next cell in the current direction;
    - if the current cell contains the letter C, write this letter down on the paper, face south, and move to the next cell to the south;
    - if the current cell contains the letter M, write this letter down on the paper, face east, and move to the next cell to the east;
    - if the current cell contains the letter W, write this letter down on the paper, face north, and move to the next cell to the north.

    Natalie wants to place the five letters C, M, W, M, C on distinct cells. Compute the number of such placements that will cause the robot to eventually reach the end cell and, upon reaching it, to have written exactly the sequence C, M, W, M, C in that order. (Below is one example placement.)



18. Let m and n be positive integers. If $\dfrac{n^5 + m}{n - 1}$ is also a positive integer, find the smallest m such that there are 12 solutions for n.

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19. Given a triangle ABC, let $AB = 7$, $BC = 8$, and $CA = 9$. Let E be the point on circle ABC not equal to A such that $AE \perp BC$. Express $CE = \frac{m}{\sqrt{n}}$, where m and n are positive integers with n squarefree. Find $m + n$.

20. Denote $\overline{z}$ as the complex conjugate of z. Let z_1 and z_2 be complex numbers such that

$$(z_1\overline{z_2}) + (\overline{z_1}z_2) = 32$$

and

$$|z_1\overline{z_2}| \cdot |\overline{z_1}z_2| = 300.$$

If z_1 and z_2 represent points A and B on the complex plane with origin O, compute the area of $\triangle AOB$.

21. Compute the sum of all positive integers m such that

$$\sum_{k=1}^{1000} \left\lfloor \frac{km}{1000} \right\rfloor = 500m.$$