

Problem 1

CMWMC 2024, Relay Round, Set 1/4

The sum of the digits of positive integer N is 55. Find the least possible sum of the digits of $N + 111$.

Problem 2

CMWMC 2024, Relay Round, Set 1/4

Let T denote the answer to the previous problem. There exists a unique ordered pair (a, b) of positive integers with $b < T$ and

$$a + \frac{b}{T} = \frac{3}{2} \times \frac{ab}{T}.$$

What is ab ?

Problem 3

CMWMC 2024, Relay Round, Set 1/4

Define the *middle common factor* of three positive integers a, b, c to be

$$\text{mcf}(a, b, c) = \frac{abc}{\gcd(a, b, c) \cdot \text{lcm}(a, b, c)}.$$

For example, $\text{mcf}(10, 12, 40) = \frac{4800}{2 \cdot 120} = 20$.

Let T denote the answer to the previous problem. As n ranges over the positive integers, find the number of possible values for $\text{mcf}(40, T, n)$.

Problem 1

CMWMC 2024, Relay Round, Set 2/4

Evaluate

$$\sum_{k=0}^{2024} \sin\left(\frac{3\pi}{2} + \frac{k\pi}{4}\right).$$

Problem 2

CMWMC 2024, Relay Round, Set 2/4

Let T denote the answer to the previous problem. The cubic equation

$$x^3 - Tx^2 = 1 - T$$

has three distinct (real or complex) solutions r_1, r_2, r_3 . Find $|r_1|^2 + |r_2|^2 + |r_3|^2$.

Problem 3

CMWMC 2024, Relay Round, Set 2/4

Let T denote the answer to the previous problem. The tax rate in the United States of Bmerica (USB) is $r = \frac{1}{2T}$, i.e. whenever anyone receives an asset of value d , they must pay rd dollars in tax. Irene and Robin live in the USB and Irene wants to give Robin a car, worth \$63,720, as a gift. But she knows Robin must pay taxes on that car, so she also wants to give Robin some money s to pay for the tax. But such a gift will be taxed as well, so Irene needs to give more money to cover the tax on s , and so on ad infinitum.

How much money s should Irene give Robin to exactly cover all of the tax (i.e. the tax on the car and also the tax on all the money Irene is giving Robin)?

Problem 1**CMWMC 2024, Relay Round, Set 3/4**

How many permutations of the word “CMWMC” are there such that there are an odd number of letters between the two C’s?

Problem 2**CMWMC 2024, Relay Round, Set 3/4**

Let T denote the answer to the previous problem. Let X denote the number different ways are there to color the edges of a square with T colors such that no two adjacent edges are the same color. What is the value of $\lfloor \frac{X}{2220} \rfloor$? Assume a fixed orientation of the square, i.e. rotations of a coloring are considered different colorings.

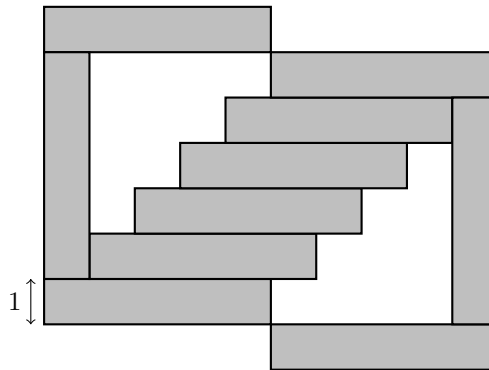
Problem 3**CMWMC 2024, Relay Round, Set 3/4**

Let T denote the answer to the previous problem. Michael is playing the hot new game, Quadramon. Michael wants to collect a team of monsters, which happens through “encounters”. For every encounter, Michael has probability $\frac{9}{16}$ of adding a monster to their team, $\frac{7}{16} - \frac{1}{T-2}$ of not adding a monster to their team, and $\frac{1}{T-2}$ of losing all their monsters and having to quit the game. Michael can stop whenever he wants, and can perform as many encounters as they want until they lose all their monsters. If Michael wants to maximize the expected size of his team, there is a unique strategy that accomplishes this. If the expected number of encounters it takes to execute the strategy is a fraction $\frac{a}{b}$, where a, b are positive coprime integers, find $a + b$.

Problem 1

CMWMC 2024, Relay Round, Set 4/4

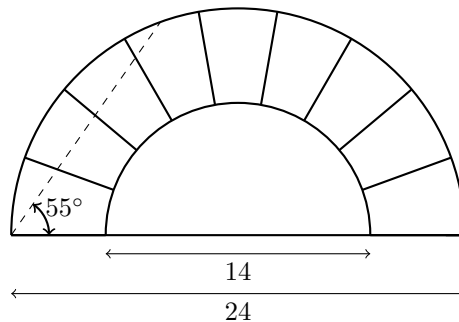
All rectangles in the below figure are congruent. What is the area of the shaded region?

**Problem 2**

CMWMC 2024, Relay Round, Set 4/4

Let T denote the answer to the previous problem. Hamerschlag Hall is renovating its windows to a new design. The window shape is a semicircle of diameter 24 inches, but is constructed by gluing many panes. Specifically, it uses $T + 1$ panes: one semicircular pane of diameter 14 inches, and T congruent panes that fill in the area outside the semicircular pane.

While no one was looking, a ninja uses her sword to make a straight line cut through a renovated window such that it makes a 55° angle with the horizontal and cuts through the window corner, thus damaging a bunch of panes. The total area of panes that need to be replaced is $k\pi$. Find k . (Below is an example window with $T = 9$. In this case, four panes need to be replaced since the sword cut goes through four panes).

**Problem 3**

CMWMC 2024, Relay Round, Set 4/4

Let T denote the answer to the previous problem. A quadrilateral $ABCD$ has $AB = 17$, $BC = 27$, $CD = 15$, and $DA = T$. $ABCD$ has exactly two right angles. What is the area of $ABCD$?