



2024 Individual Round

Instructions

1. Do not look at the test before the proctor starts the round.
2. This test consists of 20 short-answer problems to be solved in 60 minutes. The final estimation question will be used to break ties.
3. No computational aids other than pencil/pen are permitted.
4. Write your name, team name, and team ID on your answer sheet.
5. Write your answers in the corresponding lines on your answer sheet.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest on the CMWMC discord channel.



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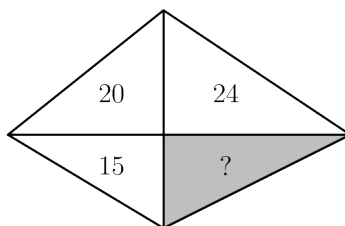


Individual Round

1. Compute

$$\frac{1}{2} \left(1 + \frac{1}{2} \left(1 + \frac{1}{2} \left(1 + \frac{1}{2} \left(1 + \frac{1}{2} \left(1 + \frac{1}{2} \right) \right) \right) \right) \right) \right).$$

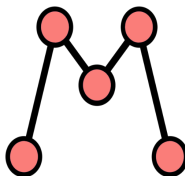
2. Let p, q, r be primes with $p + q + r = 26$. What is the maximum possible value of pqr ?
3. In the diagram below, the areas of three triangles are given. What is the area of the shaded triangle?



4. Shyla has five identical marbles. On two of them, she writes a “C”, and on the other three, she writes an “M”. She puts all five marbles in a bag, and shuffles them up. She then pulls out the five marbles one-by-one in a random order. If Shyla pulls a marble with an “M,” she can choose to turn it upside-down to get a “W”.

What is the probability that she pulls the marbles in an order that can be interpreted as C, M, W, M, C?

5. Find the remainder when 7^7 is divided by 100.
6. Consider the M of the CMIMC logo. Imagine that you play a game: you start at the middle node. On each turn, you have a 50% chance of moving to either neighbor. However, if at any time you only have one neighbor, the game ends.



What is the probability that the game is still going after ten moves?

7. Charlotte travels at a constant speed of 2 miles per hour up a mountain, and at a constant speed of 5 miles per hour back down. Written as a simplified fraction, what is Charlotte’s average speed over the whole trip?
8. Jenny is trying to take a photo of the CMIMC club members. They first try to put everyone in rows of 5, but there are 2 people left over. Then they try to put everyone in rows of 7, but there are 3 people left over. Then they try to put everyone in rows of 9, but there are 4 people left over. Given that there are fewer than 300 people in CMIMC, how many club members are there (excluding the photographer)?
9. Simplify the following fraction:

$$\frac{1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + 13 + 14 + \cdots + 298 + 299}{3 + 6 + 9 + 12 + 15 + 18 + \cdots + 297 + 300}$$

10. How many ways can we arrange the numbers 1, 1, 2, 2, 3, 3 such that 1 is not in the third position?

11. Let ABC be a triangle with $AB = 5$, $BC = 12$, $AC = 13$. We put points D_1, D_2 on AB , points E_1, E_2 on BC , and points F_1, F_2 on AC such that:

- $AD_1 = BD_2 = \frac{5}{3}$,
- $BE_1 = CE_2 = 4$,
- $AF_1 = CF_2 = \frac{13}{3}$.

Find the area of the hexagon $D_1D_2E_1E_2F_1F_2$.

12. The polynomials

$$P(x) = x^3 - 4x^2 - 25, \quad Q(x) = x^3 - 8x^2 + 11x + 20$$

share a common root. What is this root?

13. What is the smallest positive integer for which the product of its factors is a multiple of 2024?
(For example, the product of the factors of 12 is

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot 6 \cdot 12 = 1728,$$

but this is not a multiple of 2024.)

14. A parallelogram has sides of length 13 and 15. If one diagonal of this parallelogram has length 14, find the length of the other diagonal.

15. Sharon is playing a variant of chess where the initial pieces on the first row are shuffled. There are 1 king, 1 queen, 2 knights, 2 rooks, and 2 bishops, and pieces of the same type are indistinguishable from one another. Furthermore, bishops must be on opposite-colored squares, and both sides have the same configuration. How many possible starting positions are there?

16. Claire digs a circular hole of radius $r < 4$ into perfectly-flat ground. If she places a sphere of radius 4 into the hole, its lowest point sits at some distance d below the surface. If she instead places a sphere of radius 8 into the hole, its lowest point sits at a distance $d - 2$ below the surface.

Compute the radius of a sphere such that, if it were placed into the hole, its center would sit at a distance of $d - 1$ below the surface.

17. For $x \in \{0, 1, 2, \dots, 10000\}$, let

$$f(x) = \left\lfloor \frac{x^2}{2024} \right\rfloor.$$

How many distinct values does f take as x ranges over $\{0, 1, 2, \dots, 10000\}$?

18. Find the greatest common divisor of F_{555} and F_{550} , where F_n is the n th term in the Fibonacci sequence with $F_1 = F_2 = 1$.

19. Given a triangle $\triangle ABC$ such that $AB = 24$, $BC = 32$, and $AC = 40$, let D be a point on BC such that $BD = 7$. Furthermore, let N be the incenter of $\triangle ABC$ and M the incenter of $\triangle ABD$.

What is the area of $\triangle NDM$?

20. Suppose a, b, c, d are uniformly chosen divisors of 6^5 . How many possible ways are there to choose such a, b, c, d such that

$$a \mid b, \quad a \mid c, \quad b \nmid c, \quad c \nmid b, \quad c \mid d, \quad \text{and} \quad b \mid d?$$

Note $a \mid b$ is shorthand for a divides b . For example, $2 \mid 12$.

21. (Estimation) Nine points are placed in the plane such that no three are collinear. Estimate the minimum number of convex quadrilaterals which can possibly be formed by connecting four of the points.