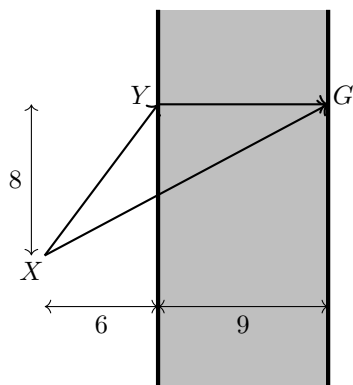


CMWMC 2024, Guts Round, Set 1/8

1. Jesse the ant is on one vertex of a cube. How many paths are there for her to get from her vertex to the opposite vertex, given that she travels on just the edges and reaches the other side in 3 moves?
2. There is a straight river 9 furlongs in width, and a spotted lanternfly 6 furlongs away from the left bank of the river (point X in the diagram). The lanternfly wants to reach a grapevine on the right bank of the river (point G), 8 furlongs downstream. The lanternfly could go directly to the grapes, travelling A furlongs. Alternatively the lanternfly could go to the point on the left bank straight across from the grapes (point Y), then cross the river, travelling a total of B furlongs. Find $|B - A|$.



3. A bacterial colony doubles in size every n minutes, for some n . However it is quite shy and pauses growth for 30 minutes after being observed. Laura observes the colony at 1:00 PM and records a size of 1 mm^2 . Laura observes the colony again at 2:50 PM and records a size of 16 mm^2 . What will the size of the colony (in mm^2) be when Laura checks again at 4:20 PM?

CMWMC 2024, Guts Round, Set 2/8

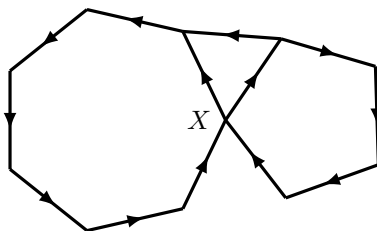
4. Consider the regular hexagon $ABCDEF$ with side length 1, and consider points $X_1, X_2, X_3, \dots, X_{2024}$ which all lie in the interior of the hexagon. Find the minimum value of

$$AX_1 + X_1X_2 + X_2X_3 + \dots + X_{2023}X_{2024} + X_{2024}C.$$

5. At a bakery, Christina crumbles one out of every five cookies that she individually packages. Cookie crumbling occurrences are independent from one another. What is the probability that the third cookie she crumbles occurs on the fifth instance of packaging a cookie?
6. How many 3-digit numbers $\underline{ABC}$ are such that $A + C - B$ is a multiple of 11?

CMWMC 2024, Guts Round, Set 3/8

7. Find the sum of the values of x such that $|||x - 1| - 1| - 1| = 1$.
8. How many ways can you tile a 3×6 board with 6 L-shaped triominoes (a 2×2 square without one tile)?
9. Nicole goes for a walk on a clear, moonlight, midnight that starts from her house at point X . At each minute $t = 0, 1, 2, \dots$ she walks along a directed arrow in the graph. What is the largest value of t for which Nicole cannot possibly be back at X ? For example, at time step 3 she cannot be back home because there's no loop of length 3. She also can't be back at time 9, even though she's passed her house, because she'd have to walk additional steps.



CMWMC 2024, Guts Round, Set 4/8

10. Let $p \in \mathbb{R}$ such that the polynomial $x^2 + px + \frac{p^2}{2}$ has two (not necessarily distinct) real roots a, b . Compute

$$\frac{(a - p + 2024)(b - p + 2024)}{2024 - ab}.$$

11. What is the greatest integer n such that 2^n divides

$$1 \times (1 + 2) \times (1 + 2 + 3) \times \cdots \times (1 + 2 + \cdots + 100)?$$

12. We have 3 distinct points on the plane X, Y, Z , with $a = XY, b = XZ$, and $c = YZ$ such that $a \geq b \geq c$. Define the obtuseness of these points to be $\min(\frac{a}{b}, \frac{b}{c})$. What is the maximum possible obtuseness of 3 points?

CMWMC 2024, Guts Round, Set 5/8

13. How many integers are roots of

$$(x^2 - x)(x^2 - x - 1)(x^2 - x - 2) \cdots (x^2 - x - 200)?$$

14. Find the number of nonnegative integers $N < 1000$ such that $\frac{15000 + N}{15 - N}$ is an integer.
15. A right circular cylindrical tank of height 100 and radius 10 was stood on its circular base and partially filled with water. Unfortunately, after an earthquake, it tipped over onto its long side, so we weren't able to measure the water level! We measured the water level while it was on its side, and found that the height of the water was exactly $10 - 5\sqrt{2}$. What was the height of the water in the tank before the earthquake?



CMWMC 2024, Guts Round, Set 6/8

16. Katherene and April are playing a game on a 2×2 grid of squares. On each round, both players simultaneously pick a square on the grid. If they pick the same square, they (collectively) win 3 points. Otherwise, if their squares still lie in the same row or column, they win 1 point.

If Katherene and April both pick a square uniformly at random during each round, what is their expected score at the end of 4 rounds?

17. Find the number of ordered pairs of integers (x, y, z) with $|x|, |y|, |z| \leq 10$, such that

$$|x + y + z| \text{ is prime and } (x - y)^2 + (x - z)^2 + (y - z)^2 = 2.$$

18. Let $A = (4, 1)$, $B = (10, 9)$, and $C = (9, 6)$. Draw a circle with diameter $\overline{AB}$. Construct point P on this circle such that $\angle APC = \angle BPC = 45^\circ$. Find the sum of the coordinates of P .

CMWMC 2024, Guts Round, Set 7/8

19. Find the least rational number x such that it is possible to write

$$x = 0.\overline{ABCDEF}_2 = 0.\overline{GH}_6$$

for some digits $0 \leq A, B, C, D, E, F \leq 1$ and $0 \leq G, H \leq 5$. (These are repeating “decimals” in base 2 and base 6.)

20. Four points are evenly spaced on a circle of radius 2. Four circles of equal radius are drawn centered at these points such that each circle is externally tangent to two other circles. These four circles intersect the original circle at eight points. Find the area of the convex octagon formed by connecting these points.
21. Annie has 4 wooden cubes, of side lengths 1cm, 2cm, 3cm, and 4cm, respectively. Annie then constructs a tower by stacking the cubes on top of each other, where the cube at each step is chosen uniformly at random. The tower is constructed such that the centers of the cubes are collinear to a line perpendicular to the ground. What is the expected surface area of such a tower? (Don't count the face that touches the ground.)

CMWMC 2024, Guts Round, Set 8/8

22. Find the value of $\left\lfloor \frac{44^{2024}}{44^{92} - 7} \right\rfloor \pmod{44}$.

23. Find the number of ways to tile a 2×5 board with dominoes (1×2 or 2×1) and 1×1 tiles.

24. Let $Q(x)$ be a monic polynomial such that

$$Q(x) = P(x)(x + 45) = P(x - 45)(x + 2025)$$

for some $P(x)$. Find $Q(10) \pmod{2025}$.
