Problem 1 **CMWMC 2023, Relay Round, Set 1/4**

How many positive integer divisors are there of $2^2 \cdot 3^3 \cdot 5^4$?

Problem 2 **CMWMC 2023, Relay Round, Set 1/4**

Let T be the answer from the previous problem. For how many integers n between 1 and T (inclusive) is $\frac{(n)(n-1)(n-2)}{12}$ an integer?

Problem 3 **CMWMC 2023, Relay Round, Set 1/4**

Let T be the answer from the previous problem. Find $\frac{\text{lcm}(T, 36)}{\text{gcd}(T, 36)}$.

Problem 1 **CMWMC 2023, Relay Round, Set 2/4**

A school has 50 students and four teachers. Each student has exactly one teacher, such that two teachers have 10 students each and the other two teachers have 15 students each. You survey each student in the school, asking the number of classmates they have (not including themself or the teacher). What is the average of all 50 responses?

Problem 2 **CMWMC 2023, Relay Round, Set 2/4**

Let T be the answer from the previous problem. A ball is thrown straight up from the ground, reaching (maximum) height $T+1$. Then the ball bounces on the ground and rebounds to height $T-1$. The ball continues bouncing indefinitely, and the height of each bounce is r times the height of the previous bounce for some constant r. What is the total vertical distance that the ball travels?

Problem 3 **CMWMC 2023, Relay Round, Set 2/4**

Let T be the answer from the previous problem. The polynomial equation

$$x^3 + x^2 - (T+1)x + (T-1) = 0$$

has one (integer) solution for x which does not depend on T and two solutions for x which do depend on T. Find the greatest solution for x in this equation. (Hint: Find the independent solution for x while you wait for T.)

Problem 1 **CMWMC 2023, Relay Round, Set 3/4**

Find the number of distinct values that can be made by inserting parentheses into the expression

$$1-1-1-1-1-1$$

such that you don't introduce any multiplication. For example, $(1-1)-((1-1)-1-1)$ is a valid way to insert parentheses, but $1-1(-1-1)-1-1$ is not.

Problem 2 **CMWMC 2023, Relay Round, Set 3/4**

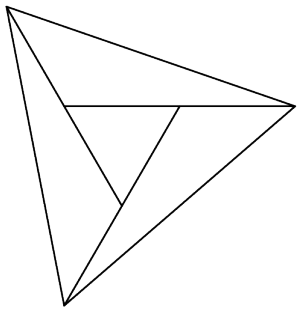
Let T be the answer from the previous problem. Katie rolls T fair 4-sided dice with faces labeled 0-3. Considering all possible sums of these rolls, there are two sums that have the highest probability of occurring. Find the smaller of these two sums.

Problem 3 **CMWMC 2023, Relay Round, Set 3/4**

Let T be the answer from the previous problem. Amy has a fair coin that she will repeatedly flip until her total number of heads is strictly greater than her total number of tails. Find the probability she will flip the coin exactly T times. (Hint: Finding a general formula in terms of T is hard; try solving some small cases while you wait for T.)

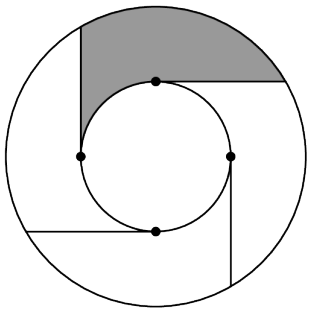
Problem 1 **CMWMC 2023, Relay Round, Set 4/4**

Triangle T has side lengths 1, 2, and $\sqrt{7}$. It turns out that one can arrange three copies of triangle T to form two equilateral triangles, one inside the other, as shown below. Compute the ratio of the area of the outer equilaterial triangle to the area of the inner equilateral triangle.



Problem 2 **CMWMC 2023, Relay Round, Set 4/4**

Let T be the answer from the previous problem. The diagram below features two concentric circles of radius 1 and T (not necessarily to scale). Four equally spaced points are chosen on the smaller circle, and rays are drawn from these points to the larger circle such that all of the rays are tangent to the smaller circle and no two rays intersect. If the area of the shaded region can be expressed as $k\pi$ for some integer k, find k.



Problem 3 **CMWMC 2023, Relay Round, Set 4/4**

Let T be the answer from the previous problem. T^2 congruent squares are arranged in the configuration below (shown for $T = 3$), where the squares are tilted in alternating fashion such that they form congruent rhombuses between them. If all of the rhombuses have long diagonal twice the length of their short diagonal, compute the ratio of the total area of all of the rhombuses to the total area of all of the squares. (Hint: Rather than waiting for T, consider small cases and try to find a general formula in terms of T; such a formula does exist.)