CMIMC

Team Round

Instructions

1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 40 minutes. The final estimation question will be used to break ties.
3. No computational aids other than pencil/pen are permitted. You are allowed to communicate with your team members.
4. Write your name, team name, and exactly one competitor ID. It does not matter which team member writes their competitor ID on your answer sheet.
5. Write your answers in the corresponding lines on your answer sheet.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest to the 2026 CMIMC Discord.
8. Suspicion of cheating may render disqualification of scores and results.

Jane Street
Jump Trading
FIVE RINGS
HRT hudson river trading
CITADEL | CITADEL Securities
DEShaw&Co
Tower Research Capital
TWO SIGMA
SUSQUEHANNA

Team

1. Equiangular hexagon $NARUTO$ has sides $NA = 5$, $AR = 4$, $RU = 1$, $UT = 7$, $TO = 2$, and $ON = 3$. If the area of $NARUTO$ can be expressed as $\frac{a\sqrt{b}}{c}$ in simplest form, find the value of $a + b + c$.

2. Jenny draws a 2×20 lattice. She picks one lattice point to be red and a different lattice point to be blue. And then she challenges Sam to draw a path from red to blue, where each segment of the path is 1 unit long and in cardinal directions, and going through each lattice point exactly once (excluding the start and end points). For how many choices of red and blue lattice points is it possible for Sam to succeed.

3. James runs a business making customized helmets, and recently received a commission from Michael. Michael's head shape is quite strange however, so he sent James a front, side, and top down view of his head. In each of the three images, Michael's head appears to be a circle of radius 1. Given that James only makes spherical helmets, he is wondering: What is the minimum R such that Michael's head is guaranteed to fit in a sphere of radius R?

4. Find the greatest ϵ such that $f(n) = \frac{\sigma(n)}{n^\epsilon}$ is maximized at some n divisible by 2026, where $\sigma(n)$ is the number of positive factors of n.

5. There is a unique nonzero complex number z such that the system of equations $|w - 8| = 5$, $|w + 2 + 5i| = 5\sqrt{2}$, $|w| = |w - z|$ has more than one solution in w. Find z.

6. The superfactorial of a positive integer n is $F(n) = 1! \cdot 2! \cdot 3! \cdot \cdots \cdot n!$.

   Find the product of all positive integers $n \leq 20$ such that there exists a positive integer m where $\frac{F(n)}{m!}$ and $\frac{F(n)}{(m+1)!}$ are both perfect squares.

7. An 8-sided die is crafted, and the probability p_i of the i-th face being rolled is generated as follows: p_1 is a uniformly random number from 0 to 1, p_k is a uniformly random number from 0 to $1 - \sum_{i=1}^{k-1} p_i)$ for all k from 2 to 7, with the p_k generated iteratively in order. The 8th side has all the leftover probability; $p_8 = 1 - \sum_{i=1}^{7} p_i$. This die is then rolled 8 times. Find the probability that in these 8 rolls, each face from 1 to 8 shows up exactly once.

8. The products in an $n \times n$ multiplication table are summed up to obtain S. Find the smallest $n > 1$ such that S has exactly n factors.

9. Let $\triangle ABC$ be a right triangle with $AB = 33, BC = 44, AC = 55$. Let X, Y be on the sides AB, BC respectively such that $AX = 11, CY = 11$. Let AY intersect CX at P. Then, let E, F be on AX, CY such that $XE = CF$. Then, let EF intersect AY, CX at the points R and S respectively. Find the minimum circumradius of the triangle $\triangle PRS$.

10. Suppose $P(x)$ is a polynomial of degree 2024 such that for all $0 \leq n \leq 2024$, we have $P(n) = F_n$. (Here, F_n is the Fibonacci sequence given by $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.) Find $P(2025)$.

11. **(Tiebreaker)** Let $F_{1000000}$ be the one millionth Fibonacci number in base 10. Let $f(n)$ for a positive integer n denote how many maximal strictly increasing subsequences we can partition n into. For example,

$$f(135246) = 2$$

because there are two maximal increasing sequences, 135 and 246. Another example to consider is:

$$f(74382) = 4$$

because there are 4 maximal increasing sequences, $7, 4, 38, 2$. Estimate the value of:

$$f(F_{1000000})$$