Geometry Round

Instructions

1. Do not look at the test before the proctor starts the round.
2. This test consists of 8 short-answer problems to be solved in 50 minutes. The final estimation question will be used to break ties
3. No computational aids other than pencil/pen are permitted. You may NOT communicate with your teammates.
4. Write your name, team name, and competitor ID on your answer sheet.
5. Write your answers in the corresponding lines on your answer sheet.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest to the 2026 CMIMC Discord.
8. Suspicion of cheating may render disqualification of scores and results.

Geometry

1. Consider triangle $\triangle ABC$ with side lengths 6, 7, and 10. Sebastian places a point P somewhere in the plane and calculates $\max(AP, BP, CP)$. What is the minimum possible number he could have calculated?

2. Two spheres are externally tangent to each other, one with radius 7 and one with radius 5. What is the side length of the smallest cube that fully contains both?

3. Non-degenerate triangle ABC has side lengths $n, n+1, n+2$, where n is an integer. Interestingly, the median altitude of this triangle is also an integer. Find the sum of the three lowest possible n.

4. A cone has a circular base with a radius of 4. Its vertex is 6 units directly above the base, but it is offset from the center axis by 2 units. As the vertex rotates in a full circle around that center point (maintaining its height of 6), the slanted cone sweeps through space. What is the total volume of the solid formed by this motion?

5. Let $\triangle ABC$ be an isosceles triangle with $AB = AC$. Let $D \neq B$ be the point on line $\overline{BC}$ such that the circle passing through A, B, and D is tangent to line $\overline{AC}$ at A. Let E be the point on line $\overline{BC}$ such that $AE \perp AC$. Given that $CD = 30$ and $AE = 3AC$, compute the area of $\triangle ADE$.

6. Let $ABCD$ be a convex quadrilateral such that $AB = 8$, $BC = 27$, $CD = 36$. Let M and N be the midpoints of AC, BD respectively. Suppose that the angle bisectors of $\angle CBA, \angle DAB$ intersect at a point L, and that L lies on MN. Finally, suppose that $\cos(\angle CDA) = \frac{11}{23}$. Find the area of $\triangle LAB$.

7. Let M, B, C, A be points such that $MC = 21$. Let X, Y be points chosen such that X is on the same side of BC as M, Y is on the opposite side, MX is minimized, MY is maximized and that the following two angle relations hold:

$$\angle XBM - \angle XCA + \angle YBM + \angle YCA = 90^\circ$$

$$\angle YBM + \angle YCA - \angle XBM + \angle XCA + 360^\circ = 2(\angle BMC + \angle MCA)$$

   Let $MX = 9$. It is known that $\angle CYA = \angle XCM = 1/2\angle CAX$ and that $AX = AY$. Find the area of $\triangle MCY$.

8. In the diagram below, four of the five drawn circles have radii marked as 12, 16, 28, and r. Find r.

   (Circles in the diagram are tangent to the lines, but they are not necessarily tangent to each other, even if they seem to be.)

9. **(Tiebreaker)** We choose 4 points in the space $[0,1]^3 \subseteq \mathbb{R}^3$. The circumradius of the sphere on which those 4 points lie is r. Find the expected value of $\frac{1}{r^2+1}$.