Algebra and Number Theory Round

Instructions

1. Do not look at the test before the proctor starts the round.
2. This test consists of 8 short-answer problems to be solved in 50 minutes. The final estimation question will be used to break ties.
3. No computational aids other than pencil/pen are permitted. You may NOT communicate with your teammates.
4. Write your name, team name, and competitor ID on your answer sheet.
5. Write your answers in the corresponding lines on your answer sheet.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest to the 2026 CMIMC Discord.
8. Suspicion of cheating may render disqualification of scores and results.

Algebra and Number Theory

1. You are handing out candies to Alice, Bob, and Carol, and they each receive $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ of the total number of candies, respectively. Afterwards, you realize the leftover candies can be split evenly between Alice, Bob, Carol, and yourself. What's the lowest number of candies you could've started with?

2. The average temperature in Pittsburgh, PA over the past 12 days was $23°F$. If no temperature occurred more than twice, every temperature was an integer, and the minimum temperature was $15°F$, find the maximum possible temperature in any of these 12 days.

3. Let k be the sum of the first $2^{89} - 1$ positive integers. It is a fact that k has 178 factors. If n is the sum of the first $2^{89} - 1$ positive cubes, how many factors does n have?

4. Consider the arithmetic sequence $a_1, a_2, ..., a_{43}$ with common difference of $d > 0$ and $a_{22} = 1$. Given that

$$\frac{1}{a_1 \cdot a_2 \cdot a_3} + \frac{1}{a_2 \cdot a_3 \cdot a_4} + ... + \frac{1}{a_{41} \cdot a_{42} \cdot a_{43}} = 41,$$

   compute d.

5. Find the largest prime p such that $\sqrt{1024 + 3p}$ is an integer.

6. How many multiples of 3 are in the 2026th row of Pascal's Triangle, i.e. the row with 2027 entries? (We number the rows starting from 0.)

7. Consider the solutions to

$$3x^2 + 4xy + y^2 + 2x + 4y \equiv 1 \mod 83$$

   where x, y are nonnegative integers less than 83. For how many values of x does there exist a solution for y?

8. Compute $\tan^4(20°) + \tan^4(40°) + \tan^4(60°) + \tan^4(80°)$.

9. **(Tiebreaker)** For a positive integer n, define $f(n)$ to be the sum of the positive factors of n. Estimate $f(1) + \cdots + f(10,000)$.