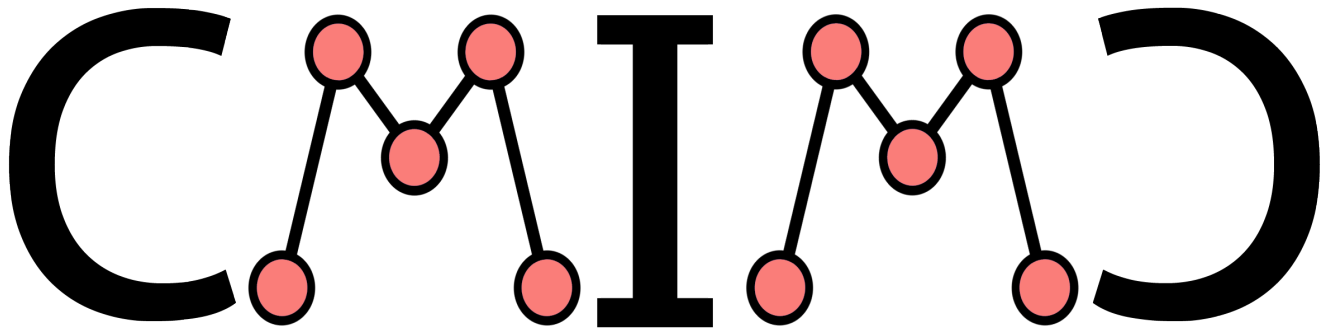




Team Round

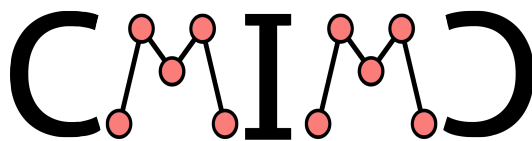
Instructions

1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 40 minutes.
3. No computational aids other than pencil/pen are permitted.
4. Write your team name on your answer sheet.
5. Write your answers in the corresponding lines on your answer sheet.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest to the 2025 CMIMC discord.



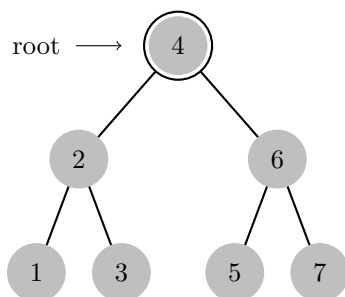
CITADEL | CITADEL Securities





Team

1. Define a “good day” as a day when both the day and the month evenly divide the concatenation of the two. For example, today (March 15) is a good day since 3 and 15 both divide 315. However, March 9 is not a good day since 9 does not divide 39. How many good days are in March, April, and May combined?
2. We are searching for the number 7 in the following binary tree:



We use the following algorithm (which terminates with probability 1):

- (a) Write down the number currently at the root node
- (b) If we wrote down 7, terminate
- (c) Else, pick a random edge, and swap the two numbers at the endpoints of that edge
- (d) Go back to step 1

Let $p(a)$ be the probability that we ever write down the number a after running the algorithm once. Find

$$p(1) + p(2) + p(3) + p(5) + p(6).$$

3. Let $f(x) = x^4 - 4x^2 + 2$. Find the smallest natural $n \in \mathbb{N}$ such that there exists $k, c \in \mathbb{N}$ with

$$\left| f^k \left(\frac{n^2 + 1}{n} \right) - c^{144} \right| < \frac{1}{100}.$$

Here, $f^k(x) = f(f^{k-1}(x))$ for $k \geq 2$, and $f^1(x) = f(x)$.

4. A non-self intersecting hexagon *RANDOM* is formed by assigning the labels R, A, N, D, O, M in some order to the points

$$(0, 0), (10, 0), (10, 10), (0, 10), (3, 4), (6, 2).$$

Let $a_{\max}$ and $a_{\min}$ be the greatest and least possible area of *RANDOM*, respectively. Find $a_{\max} - a_{\min}$.

5. Suppose we have a uniformly random function from $\{1, 2, 3, \dots, 25\}$ to itself. Find the expected value of

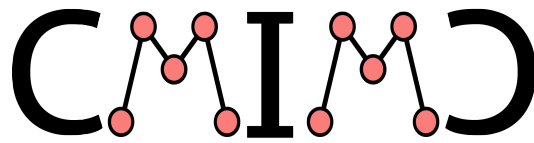
$$\sum_{x=1}^{25} (f(f(x)) - x)^2.$$

6. Suppose we have a regular 24-gon labeled $A_1 \cdots A_{24}$. We will draw 2 smaller regular 24-gons within $A_1 \cdots A_{24}$. For the sake of this problem, make $A_i = A_{i+24}$.

With our first configuration, we create 3 stars by drawing the segment $\overline{A_i A_{i+9}}$ for each $i = 1, 2, \dots, 24$. A 24-gon will be created in the center, which we denote as our first 24-gon.

With our second configuration, we create a star by drawing the segment $\overline{A_i A_{i+11}}$ for each $i = 1, 2, \dots, 24$. A 24-gon will be created in the center, which we denote as our second 24-gon.

Find the ratio of the areas of the first 24-gon to the second 24-gon.



7. The binomial coefficient $\binom{n}{k}$ can be defined as the coefficient of x^k in the expansion of $(1+x)^n$. Similarly, define the trinomial coefficient $\binom{n}{k}_3$ as the coefficient of x^k in the expansion of $(1+x+x^2)^n$.

Determine the number of integers k with $0 \leq k \leq 4048$ such that $\binom{2024}{k}_3 \equiv 1 \pmod{3}$.

8. Let U be the set of all possible complex numbers m so that the 4 roots of $(x^2+2x+5)(x^2-2mx+25)=0$ are concyclic in the complex plane. One can show that when the points of U are plotted on the complex plane, it is visualized as the finite union of some curves. Find the sum of the length of these curves (i.e. the perimeter of U).
9. Given triangle ABC with $AB=78, BC=50, AC=112$, construct squares $ABXY, BCPQ, ACMN$ outside the triangle. Let L_1, L_2, L_3 be the midpoints of $\overline{MP}, \overline{QX}, \overline{NY}$, respectively. Find the area of $L_1L_2L_3$.
10. In a 2024×2024 grid of squares, each square is colored either black or white. An ant starts at some black square in the grid and starts walking parallel to the sides of the grid. During this walk, it can choose (not required) to turn 90° clockwise or counterclockwise if it is currently on a black square, otherwise it must continue walking in the same direction.

A coloring of the grid is called *simple* if it is **not** possible for the ant to arrive back at its starting location after some time. How many simple colorings of the grid are maximal, in the sense that adding any black square results in a coloring that is not simple?

11. **(Tiebreaker)** Give us a 8-character string (of letters and numbers) and tie is broken by the highest CRC-16 hash value.

```
# from https://stackoverflow.com/questions/10564491/function-to-calculate-a-crc16-checksum
```

```
def crc16(string):
    crc = 0
    for char in string:
        crc ^= ord(char)
        for _ in range(8):
            isOdd = crc % 2 == 1
            crc //= 2
            if isOdd:
                crc ^= 0xa001
    return crc
```