CMIMC 2024 Integration Bee Answers

1. $\displaystyle\int_{-\pi}^{\pi} \sin(x)\left(\frac{\sec(x)+\csc(x)}{x^{-2}}\right)\,\mathrm{d}x = \boxed{\frac{2\pi^3}{3}}$
2. $\displaystyle\int_{1}^{4}\prod_{i=0}^{\infty}\sqrt[2^i]{x}\,\mathrm{d}x = \boxed{21}$
3. $\displaystyle\int_{e^e}^{e^{e^e}} \frac{1}{\log_{\log(x)}(x^x)}\,\mathrm{d}x = \boxed{\frac{e^2-1}{2}}$
4. $\displaystyle\int_{0}^{1} \mathrm{bi}_{2025}(x)\,\mathrm{d}x = \boxed{\frac{2025}{2}}$
5. $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\sin(3x)}{\sin^3(x)}+\frac{\cos(3x)}{\cos^3(x)}\,\mathrm{d}x = \boxed{4\sqrt{3}-6}$
6. $\displaystyle\int_{-2025}^{2025} \left\lceil x-\frac{1}{2}\right\rceil \cdot \left\lceil \frac{1}{x}-\frac{1}{2}\right\rceil\,\mathrm{d}x = \boxed{\frac{13}{3}}$
7. $\displaystyle\int_{0}^{1} 4^{\sqrt[4]{x}-1}\,\mathrm{d}x = \boxed{\frac{4}{\log(4)}-\frac{12}{\log^2(4)}+\frac{24}{\log^3(4)}-\frac{18}{\log^4(4)}}$
8. $\displaystyle\int_{1}^{1+\varphi} (x^2-1)\sum_{k=0}^{100}\binom{100}{k}x^{2k-102}\,\mathrm{d}x = \boxed{\frac{3^{101}-2^{101}}{101}}$
9. $\displaystyle\int_{0}^{1}\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n |2\{4^n x\}-1|\,\mathrm{d}x = \boxed{2}$
10. $\displaystyle\int_{0}^{\infty}\sum_{k=1}^{2025}\frac{1}{(k^2+k)x^2+(2k+1)x+1}\,\mathrm{d}x = \boxed{\log(2026)}$
11. $\displaystyle\int_{0}^{2}\frac{x}{3\lambda(x)^2-x}\,\mathrm{d}x = \boxed{\frac{1}{2}\varphi}$ or $\boxed{\frac{\varphi^2-1}{2}}$ or $\boxed{\frac{1+\sqrt{5}}{4}}$
12. $\displaystyle\int_{0}^{1}\frac{x^2-2\log(x)-1}{\log^2(x)}\,\mathrm{d}x = \boxed{3\log(3)-2}$
13. $\displaystyle\int_{0}^{1/\sqrt{3}}\frac{x}{1-x^4}\left[\log(1+x^2)-\log(1-x^2)\right]\,\mathrm{d}x = \boxed{\frac{1}{8}\log^2(2)}$
14. $\displaystyle\int_{-\infty}^{\infty} 1-\cos\left(\frac{1}{x-1}+\frac{1}{x+1}\right)\,\mathrm{d}x = \boxed{2\pi}$
15. $\displaystyle\int_{0}^{\pi/2}\tan(x)^{4/5}\,\mathrm{d}x = \boxed{\frac{2\pi}{\sqrt{5}-1}}$ or $\boxed{\varphi\pi}$