CMIMC

Geometry Round

Instructions

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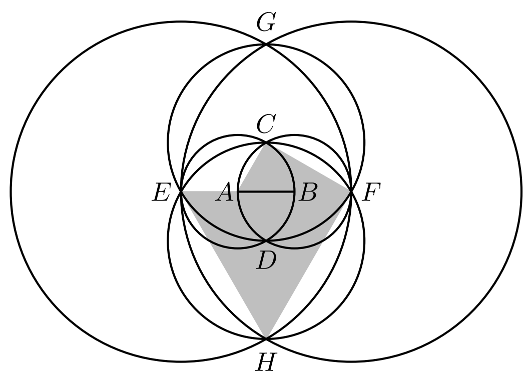
1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 50 minutes. The final estimation question will be used to break ties.
3. No computational aids other than pencil/pen are permitted.
4. Write your name and team name on your answer sheet.
5. Write your answers in the corresponding lines on your answer sheet.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest to the 2025 CMIMC discord.

Geometry

1. I'm given a square of side length 7, and I want to make a regular tetrahedron from it. Specifically, my strategy is to cut out a net. If I cut out a parallelogram-shaped net that yields the biggest regular tetrahedron, what is the surface area of the resulting tetrahedron?

2. Given a cube of side length 4, place eight spheres of radius 1 inside the cube so that each sphere is externally tangent to three others. What is the radius of the largest sphere contained inside the cube which is externally tangent to all eight?

3. Let AB be a segment of length 1. Let $\odot A$, $\odot B$ be circles with radius $\overline{AB}$ centered at A, B. Denote their intersection points C, D. Draw circles $\odot C, \odot D$ with radius $\overline{CD}$. Denote the intersection points of $\odot C$ and $\odot D$ by E, F. Draw circles $\odot E$, $\odot F$ with radius $\overline{EF}$ and denote their intersection points G, H.

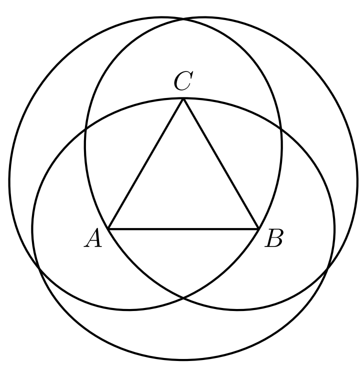


Compute the area of the pentagon $ACFHE$.

4. Let $ABCDEF$ be a regular hexagon with side length 1, and let G be the midpoint of side $\overline{CD}$, and define H to be the unique point on side $\overline{DE}$ such that $AGHF$ is a trapezoid. Find the length of the altitude dropped from point H to $\overline{AG}$.

5. Let $\triangle ABC$ be an equilateral triangle. Let E_{AB} be the ellipse with foci A, B passing through C, and in the parallel manner define E_{BC}, E_{AC}. Let $\triangle GHI$ be a (nondegenerate) triangle with vertices where two ellipses intersect such that the edges of $\triangle GHI$ do not intersect those of $\triangle ABC$.

   Compute the ratio of the largest sides of $\triangle GHI$ and $\triangle ABC$.



6. Points A, B, C, D, E, and F lie on a sphere with center O and radius R such that $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are pairwise perpendicular and all meet at a point X inside the sphere. If $AX = 1$, $CX = \sqrt{2}$, $EX = 2$, and $OX = \frac{\sqrt{2}}{2}$, compute the sum of all possible values of R^2.

7. Let ABC be a triangle with altitude $\overline{AF}$. Let $AB = 5, AC = 8, BC = 7$. Let P be on $\overline{AF}$ such that it lies between A and F. Let ω_1, ω_2 be the circumcircles of APB, APC respectively. Let $\overline{BC}$ intersect ω_1 at $B' \neq B$. Also, let $\overline{BC}$ intersect ω_2 at $C' \neq C$. Let $X \neq A$ be on ω_1 such that $B'X = B'A$. Let $Y \neq A$ be on ω_2 such that $C'A = C'Y$. Let X, Y, A all lie on one line h. Find the length of PA.

8. Let ω be a circle with diameter $\overline{AB}$, center O, and cyclic quadrilateral $ABCD$ inscribed in it, with C and D on the same side of $\overline{AB}$. Let $AB = 20$, $BC = 13$, $AD = 7$. Let $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$ intersect at E. Let the E-excircle of ECD have its center at L. Find OL.

9. Define the *ratio* of an ellipse to be the length of the major axis divided by the length of the minor axis.

   Given a trapezoid $ABCD$ with $AB \parallel DC$ and that $\angle ADC$ is a right angle, with $AB = 18, AD = 33, CD = 130$, find the smallest ratio of any ellipse that goes through all vertices of $ABCD$.

10. Let $\triangle ABC$ exist such that $AB = 6, BC = 8, AC = 10$. Let P lie on the circumcircle of ABC, ω, such that P lies strictly on the arc in between B and C (i.e. $P \neq B, C$). Drop altitudes from P to BC, AC at points J and Q respectively. Let l be a line through B such that it intersects AC at a point K. Let M be the midpoint of BQ. Let CM intersect line l at a point I. Let AI intersect JQ at a point U. Now, B, J, U, M are cyclic. Now, let $\angle QJC = \theta$. If we set $y = \sin(\theta), x = \cos(\theta)$, they satisfy the equation:

$$768(xy) = (16 - 8x^2 + 6xy)(x^2y^2(8x - 6y)^2 + (8x - 8xy^2 + 6y^3)^2)$$

    The numeric values of s, c are approximately:

$$x = 0.72951, y = 0.68400$$

    Let BK intersect the circumcircle of ABC, ω at a point L. Find the value of BL. We will only look up to two decimal places for correctness.

11. **(Tiebreaker)** We have a point P, and points $A_0, A_1, \cdots$ that such that they are all distance 2 from P, they are counter clockwise around P, and there exists a circle of radius $\frac{1}{2^i}$ tangent to PA_i and PA_{i+1} at A_i and A_{i+1}. Find the limit of the angle $\angle A_0PA_n$ as n goes to ∞ (in radians). Express it as $a.bcdefg$ (round to 6 digits)