CMIMC 2022
Geometry Round

INSTRUCTIONS

1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 60 minutes and one estimation question. Each of the short-answer questions is worth points depending on its difficulty, and the estimation question will be used to break ties. **If you do not write an estimate for estimation, you will be placed last in tiebreaking.**
3. Write your name, team name, and team ID on your answer sheet. Circle the subject of the test you are taking.
4. Write your answers in the corresponding boxes on the answer sheets.
5. No computational aids other than pencil/pen are permitted.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest in writing to Doherty 2302 by the end of lunch.

Geometry

1. A circle has radius 52 and center O. Points A is on the circle, and point P on $\overline{OA}$ satisfies $OP = 28$. Point Q is constructed such that $QA = QP = 15$, and point B is constructed on the circle so that Q is on $\overline{OB}$. Find QB.

2. Let ABC be an acute triangle with $\angle ABC = 60^\circ$. Suppose points D and E are on lines AB and CB, respectively, such that CDB and AEB are equilateral triangles. Given that the positive difference between the perimeters of CDB and AEB is 60 and $DE = 45$, what is the value of $AB \cdot BC$?

3. Let $\Gamma_1, \Gamma_2, \Gamma_3$ be three pairwise externally tangent circles with radii $1, 2, 3$, respectively. A circle passes through the centers of Γ_2 and Γ_3 and is externally tangent to Γ_1 at a point P. Suppose A and B are the centers of Γ_2 and Γ_3, respectively. What is the value of $\frac{PA^2}{PB^2}$?

4. Let A and B be points on circle Γ such that $AB = \sqrt{10}$. Point C is outside Γ such that $\triangle ABC$ is equilateral. Let D be a point on Γ and suppose the line through C and D intersects AB and Γ again at points E and $F \neq D$. It is given that points C, D, E, F are collinear in that order and that $CD = DE = EF$. What is the area of Γ?

5. In triangle ABC, let I, O, H be the incenter, circumcenter and orthocenter, respectively. Suppose that $AI = 11$ and $AO = AH = 13$. Find OH.

6. Let Γ_1 and Γ_2 be two circles with radii r_1 and r_2, respectively, where $r_1 > r_2$. Suppose Γ_1 and Γ_2 intersect at two distinct points A and B. A point C is selected on ray $\overrightarrow{AB}$, past B, and the tangents to Γ_1 and Γ_2 from C are marked as points P and Q, respectively. Suppose that Γ_2 passes through the center of Γ_1 and that points P, B, Q are collinear in that order, with $PB = 3$ and $QB = 2$. What is the length of AB?

7. In acute $\triangle ABC$, let I denote the incenter and suppose that line AI intersects segment BC at a point D. Given that $AI = 3, ID = 2$, and $BI^2 + CI^2 = 64$, compute BC^2.

8. Let $ABCD$ be a cyclic quadrilateral with circumcenter O. Rays $\overrightarrow{OB}$ and $\overrightarrow{DC}$ intersect at E, and rays $\overrightarrow{OC}$ and $\overrightarrow{AB}$ intersect at F. Suppose that $AE = EC = CF = 4$, and the circumcircle of ODE bisects $\overline{BF}$. Find the area of triangle ADF.