CMIMC 2019

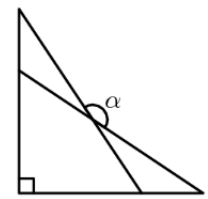
Geometry Round

INSTRUCTIONS

1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 60 minutes. Each question is worth one point.
3. Write your name, team name, and team ID on your answer sheet. Circle the subject of the test you are currently taking.
4. Write your answers in the corresponding boxes on the answer sheets.
5. No computational aids other than pencil/pen are permitted.
6. Answers must be reasonably simplified.
7. If you believe that the test contains an error, submit your protest in writing to Doherty Hall 2302 by the end of lunch.

Geometry

1. The figure to the right depicts two congruent triangles with angle measures $40°$, $50°$, and $90°$. What is the measure of the obtuse angle α formed by the hypotenuses of these two triangles?



2. Suppose X, Y, Z are collinear points in that order such that $XY = 1$ and $YZ = 3$. Let W be a point such that $YW = 5$, and define O_1 and O_2 as the circumcenters of triangles $\triangle WXY$ and $\triangle WYZ$, respectively. What is the minimum possible length of segment $\overline{O_1O_2}$?

3. Let ABC be an equilateral triangle with side length 2, and let M be the midpoint of $\overline{BC}$. Points X and Y are placed on AB and AC respectively such that $\triangle XMY$ is an isosceles right triangle with a right angle at M. What is the length of $\overline{XY}$?

4. Suppose $\mathcal{T} = A_0A_1A_2A_3$ is a tetrahedron with $\angle A_1A_0A_3 = \angle A_2A_0A_1 = \angle A_3A_0A_2 = 90°$, $A_0A_1 = 5$, $A_0A_2 = 12$ and $A_0A_3 = 9$. A cube $A_0B_0C_0D_0E_0F_0G_0H_0$ with side length s is inscribed inside $\mathcal{T}$ with $B_0 \in \overline{A_0A_1}$, $D_0 \in \overline{A_0A_2}$, $E_0 \in \overline{A_0A_3}$, and $G_0 \in \triangle A_1A_2A_3$; what is s?

5. Let $MATH$ be a trapezoid with $MA = AT = TH = 5$ and $MH = 11$. Point S is the orthocenter of $\triangle ATH$. Compute the area of quadrilateral $MASH$.

6. Let ABC be a triangle with $AB = 209$, $AC = 243$, and $\angle BAC = 60°$, and denote by N the midpoint of the major arc $\widehat{BAC}$ of circle $\odot(ABC)$. Suppose the parallel to AB through N intersects $\overline{BC}$ at a point X. Compute the ratio $\frac{BX}{XC}$.

7. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $AC = 15$. Denote by ω its incircle. A line ℓ tangent to ω intersects $\overline{AB}$ and $\overline{AC}$ at X and Y respectively. Suppose $XY = 5$. Compute the positive difference between the lengths of $\overline{AX}$ and $\overline{AY}$.

8. Consider the following three lines in the Cartesian plane:

$$\begin{cases} \ell_1: & 2x - y = 7 \\ \ell_2: & 5x + y = 42 \\ \ell_3: & x + y = 14 \end{cases}$$

and let $f_i(P)$ correspond to the reflection of the point P across ℓ_i. Suppose X and Y are points on the x and y axes, respectively, such that $f_1(f_2(f_3(X))) = Y$. Let t be the length of segment XY; what is the sum of all possible values of t^2?

9. Let $ABCD$ be a square of side length 1, and let P_1, P_2 and P_3 be points on the perimeter such that $\angle P_1P_2P_3 = 90°$ and P_1, P_2, P_3 lie on different sides of the square. As these points vary, the locus of the circumcenter of $\triangle P_1P_2P_3$ is a region $\mathcal{R}$; what is the area of $\mathcal{R}$?

10. Suppose ABC is a triangle, and define B_1 and C_1 such that $\triangle AB_1C$ and $\triangle AC_1B$ are isosceles right triangles on the exterior of $\triangle ABC$ with right angles at B_1 and C_1, respectively. Let M be the midpoint of $\overline{B_1C_1}$; if $B_1C_1 = 12$, $BM = 7$ and $CM = 11$, what is the area of $\triangle ABC$?