CMIMC 2016

Geometry Round

INSTRUCTIONS

1. Do not look at the test before the proctor starts the round.
2. This test consists of 10 short-answer problems to be solved in 60 minutes. Each question is worth one point.
3. Write your name, team name, and team ID on your answer sheet. Circle the subject of the test you are currently taking.
4. Write your answers in the corresponding boxes on the answer sheets.
5. No computational aids other than pencil/pen are permitted.
6. All answers are integers.
7. If you believe that the test contains an error, submit your protest in writing to Porter 100.

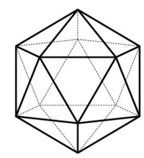
Geometry

1. Let $\triangle ABC$ be an equilateral triangle and P a point on $\overline{BC}$. If $PB = 50$ and $PC = 30$, compute PA.

2. Let $ABCD$ be an isosceles trapezoid with $AD = BC = 15$ such that the distance between its bases AB and CD is 7. Suppose further that the circles with diameters $\overline{AD}$ and $\overline{BC}$ are tangent to each other. What is the area of the trapezoid?

3. Let ABC be a triangle. The angle bisector of $\angle B$ intersects AC at point P, while the angle bisector of $\angle C$ intersects AB at a point Q. Suppose the area of $\triangle ABP$ is 27, the area of $\triangle ACQ$ is 32, and the area of $\triangle ABC$ is 72. The length of $\overline{BC}$ can be written in the form $m\sqrt{n}$ where m and n are positive integers with n as small as possible. What is $m + n$?

4. Andrew the Antelope canters along the surface of a regular icosahedron, which has twenty equilateral triangle faces and edge length 4. (An image of an icosahedron is shown to the right.) If he wants to move from one vertex to the opposite vertex, the minimum distance he must travel can be expressed as $\sqrt{n}$ for some integer n. Compute n.



5. Let $\mathcal{P}$ be a parallelepiped with side lengths x, y, and z. Suppose that the four space diagonals of $\mathcal{P}$ have lengths 15, 17, 21, and 23. Compute $x^2 + y^2 + z^2$.

6. In parallelogram $ABCD$, angles B and D are acute while angles A and C are obtuse. The perpendicular from C to AB and the perpendicular from A to BC intersect at a point P inside the parallelogram. If $PB = 700$ and $PD = 821$, what is AC?

7. Let ABC be a triangle with incenter I and incircle ω. It is given that there exist points X and Y on the circumference of ω such that $\angle BXC = \angle BYC = 90^\circ$. Suppose further that X, I, and Y are collinear. If $AB = 80$ and $AC = 97$, compute the length of BC.

8. Suppose $ABCD$ is a convex quadrilateral satisfying $AB = BC$, $AC = BD$, $\angle ABD = 80^\circ$, and $\angle CBD = 20^\circ$. What is $\angle BCD$ in degrees?

9. Let $\triangle ABC$ be a triangle with $AB = 65$, $BC = 70$, and $CA = 75$. A semicircle Γ with diameter $\overline{BC}$ is constructed outside the triangle. Suppose there exists a circle ω tangent to AB and AC and furthermore internally tangent to Γ at a point X. The length AX can be written in the form $m\sqrt{n}$ where m and n are positive integers with n not divisible by the square of any prime. Find $m + n$.

10. Let $\triangle ABC$ be a triangle with circumcircle Ω and let N be the midpoint of the major arc $\overset{\frown}{BC}$. The incircle ω of $\triangle ABC$ is tangent to AC and AB at points E and F respectively. Suppose point X is placed on the same side of EF as A such that $\triangle XEF \sim \triangle ABC$. Let NX intersect BC at a point P. Given that $AB = 15$, $BC = 16$, and $CA = 17$, compute $\frac{PX}{XN}$.